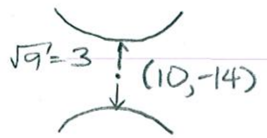
Find the co-ordinates of the vertices and foci, and the equations of the asymptotes of the hyperbola **SCORE: _____ / 5 PTS**

$\dfrac{(y+14)^2}{9} - \dfrac{(x-10)^2}{36} = 1$. **State clearly which co-ordinates are for which points.**



CENTER = $(10,-14)$

VERTICES = $(10, -14 \pm 3) = \underbrace{(10,-11)}_{1}, \underbrace{(10,-17)}_{1}$

FOCI = $\underbrace{(10, -14 \pm 3\sqrt{5})}$ 1

$c^2 = 9 + 36$

$c^2 = 45$

$c = 3\sqrt{5}$

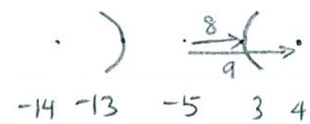
ASYMPTOTES: $m = \pm \dfrac{\sqrt{9}}{\sqrt{36}} = \pm \dfrac{3}{6} = \pm \dfrac{1}{2}$

1 $\overbrace{y + 14 = \underbrace{\pm \tfrac{1}{2}}_{1}(x-10)}$

Find the equations of the following conics. **SCORE: _____ / 9 PTS**

[a] hyperbola with vertices $(-13, 10)$ and $(3, 10)$, and foci $(4, 10)$ and $(-14, 10)$

CENTER = $\left(\dfrac{-13+3}{2}, 10\right) = (-5, 10)$



$\tfrac{1}{2}$ $\overbrace{\dfrac{(x+5)^2}{\underbrace{64}_{1}}} - \overbrace{\dfrac{(y-10)^2}{\underbrace{17}_{1\frac{1}{2}}}}^{\frac{1}{2}} \underbrace{= 1}_{\frac{1}{2}}$

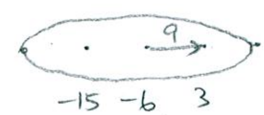
$9^2 = 8^2 + b^2$

$b^2 = 81 - 64 = 17$

+1 FOR CORRECT ORDER OF SUBTRACTION $(x^2 - y^2)$

[b] ellipse with foci $(-15, 8)$ and $(3, 8)$, and major axis of length 20

CENTER = $\left(\dfrac{-15+3}{2}, 8\right) = (-6, 8)$



$\tfrac{1}{2}$ $\overbrace{\dfrac{(x+6)^2}{\underbrace{100}_{1}}} + \overbrace{\dfrac{(y-8)^2}{\underbrace{19}_{1\frac{1}{2}}}}^{\frac{1}{2}} \underbrace{= 1}_{\frac{1}{2}}$

$2a = 20 \rightarrow a = 10$

$9^2 = 10^2 - b^2$

$b^2 = 100 - 81 = 19$

Using the distance-based definition of a hyperbola, find the equation of the hyperbola with foci $(0, \pm 8)$ such that the distances from any point on the hyperbola to the foci differ by 8. **Show the algebraic work, not just the final answer.**

SCORE: _____ / 9 PTS

IF (x,y) IS ON THE HYPERBOLA,

DISTANCE FROM (x,y) TO (0,-8) − DISTANCE FROM (x,y) TO (0,8) = ± 8

$$\sqrt{x^2+(y+8)^2} - \sqrt{x^2+(y-8)^2} = \pm 8 \quad (2)$$

$$\sqrt{x^2+(y+8)^2} = \pm 8 + \sqrt{x^2+(y-8)^2} \quad (1)$$

$$x^2+y^2+16y+64 = 64 \pm 16\sqrt{x^2+y^2-16y+64} + x^2+y^2-16y+64 \quad (2)$$

$$32y-64 = \pm 16\sqrt{x^2+y^2-16y+64}$$

$$2y-4 = \pm\sqrt{x^2+y^2-16y+64} \quad (1)$$

$$4y^2-16y+16 = x^2+y^2-16y+64 \quad (1)$$

$$3y^2-x^2 = 48 \quad (1)$$

$$\frac{y^2}{16}-\frac{x^2}{48} = 1 \quad (1)$$

Find the co-ordinates of the foci, vertices, and endpoints of the minor axis of the ellipse $4x^2+y^2+24x+10y+45=0$. **State clearly which co-ordinates are for which points.**

SCORE: _____ / 7 PTS

$$4x^2+24x+y^2+10y = -45$$

$$(1)\ 4(x^2+6x+9) + (y^2+10y+25) \left(\tfrac{1}{2}\right) = -45+4\cdot 9+25$$

$$4(x+3)^2+(y+5)^2 = 16 \quad (1)$$

$$\frac{(x+3)^2}{4}+\frac{(y+5)^2}{16} = 1 \quad \left(\tfrac{1}{2}\right)$$

$$c^2 = 16-4 = 12$$

$$c = 2\sqrt{3}$$

CENTER = $(-3,-5)$

VERTICES = $(-3,-5\pm 4) = (-3,-1), (-3,-9)$ $\left(1\tfrac{1}{2}\right)$

FOCI = $(-3,-5\pm 2\sqrt{3})$ $\left(1\tfrac{1}{2}\right)$

ENDPOINTS OF MINOR AXIS = $(-3\pm 2,-5) = (-1,-5), (-5,-5)$

(1)